## Algebraic topology <br> Problem sheet \#1 <br> Due: 16 Oct.

1. How many $i$-simplices are there in the 4 -simplex $\Delta^{4}$, for $0 \leq i \leq 4$ ? What is the fundamental group of the union of the 1-simplicies in $\bar{\Delta}^{4}$ ? What is the first homology group of that union?
2. Give an example of a nontrivial element in the fundamental group of the genus two surface whose image in the first homology group is trivial.
3. Give an example of ...
a. two spaces that are homotopy equivalent but not homeomorphic.
b. two connected spaces that have the same fundamental group but are not homotopy equivalent.
c. two spaces that have the same homology groups but are not homotopy equivalent.
4. Give a $\Delta$-complex structure on ...
a. the 2 -sphere.
b. the genus 2 surface.
c. the connected sum of three copies of $\mathbb{R P}^{2}$.
5. Compute all the homology groups of $\mathbb{R P}^{2}$ and of the Klein bottle. Redo the computation using coefficients in $\mathbb{Z} / 2$.
6. Show that if $A$ is a retract of $X$ then the map $H_{n}(A) \rightarrow H_{n}(X)$ induced by the inclusion $A \subset X$ is injective. (Hatcher 2.1.11)
7. Define the reduced homology group $\tilde{H}_{0}(X)$ as ker $p_{*}$ where $p_{*}: H_{0}(X) \rightarrow \mathbb{Z}$ is induced by the projection from $X$ to a point. Prove that $H_{0}(X) \cong \tilde{H}_{0}(X) \oplus \mathbb{Z}$.
