Algebraic topology Problem sheet #1 Due: 16 Oct.

- 1. How many *i*-simplices are there in the 4-simplex Δ^4 , for $0 \le i \le 4$? What is the fundamental group of the union of the 1-simplicies in Δ^4 ? What is the first homology group of that union?
- 2. Give an example of a nontrivial element in the fundamental group of the genus two surface whose image in the first homology group is trivial.
- 3. Give an example of ...
 - a. two spaces that are homotopy equivalent but not homeomorphic.
 - b. two connected spaces that have the same fundamental group but are not homotopy equivalent.
 - c. two spaces that have the same homology groups but are not homotopy equivalent.
- 4. Give a Δ -complex structure on ...
 - a. the 2-sphere.
 - b. the genus 2 surface.
 - c. the connected sum of three copies of $\mathbb{R}P^2$.
- 5. Compute all the homology groups of $\mathbb{R}P^2$ and of the Klein bottle. Redo the computation using coefficients in $\mathbb{Z}/2$.
- 6. Show that if A is a retract of X then the map $H_n(A) \to H_n(X)$ induced by the inclusion $A \subset X$ is injective. (Hatcher 2.1.11)
- 7. Define the reduced homology group $\tilde{H}_0(X)$ as ker p_* where $p_* : H_0(X) \to \mathbb{Z}$ is induced by the projection from X to a point. Prove that $H_0(X) \cong \tilde{H}_0(X) \oplus \mathbb{Z}$.