Algebraic topology
Problem sheet #6
Due: 28 Nov.

1. Show that every map $S^2 \to T$ induces the zero map on H_2.

2. Check that if A and B are graded commutative rings, then $A \otimes B$ is also graded commutative.

3. Compute $\mathrm{Tor}(G, H)$ for all finitely generated abelian groups G and H.

4. Compute the cohomology groups of the product of Moore spaces $M(\mathbb{Z}/n, i) \times M(\mathbb{Z}/m, j)$.

5. Show that the splitting in the homology Künneth theorem cannot be natural by considering the map $f \times \mathrm{id} : M(\mathbb{Z}/m, i) \times M(\mathbb{Z}/m, i) \to S^{i+1} \times M(\mathbb{Z}/m, i)$, where f collapses the i-skeleton of $M(\mathbb{Z}/m, i)$ to a point. (Hatcher 3.B.3)

6. Show that $H_{n-1}M$ is torsion-free for any closed orientable n-manifold M.

7. For a closed orientable $2k$-manifold M, show that if $H_{k-1}(M)$ is torsion-free then $H_k(M)$ is torsion-free. (Hatcher 3.3.25)