**Algebraic topology**
**Problem sheet #5**
**Due: 21 Nov.**

1. Use the universal coefficient theorem to compute the cohomology of $\mathbb{R}\mathrm{P}^3$ with coefficients in the quotient group $\mathbb{Q}/\mathbb{Z}$.
2. For all finitely generated abelian groups $H$, and an arbitrary abelian group $G$, compute $\mathrm{Ext}(H, G)$ in terms of $G$.
3. Let $X$ be a space obtained from $S^n$ by attaching an $(n+1)$-cell by a map of degree $m$. (This is called a Moore space and is denoted $M(\mathbb{Z}/m, n)$.) Show that the quotient $X \to X/S^n = S^{n+1}$ induces the trivial map on reduced homology, but not on cohomology. Deduce that the splitting of the universal coefficient theorem for cohomology cannot be natural. (Hatcher 3.1.11a)
4. Compute the cup product structure on the Klein bottle, with both integral and mod 2 coefficients.
5. Prove that all cup products of positive-dimensional classes in a suspension are zero. (Cf. Hatcher 3.2.2)
6. Let $X$ be $\mathbb{C}\mathrm{P}^2$ with an additional cell $e^3$ attached by a map $S^2 \to \mathbb{C}\mathrm{P}^1 \subset \mathbb{C}\mathrm{P}^2$ of degree $p$, and let $Y$ be $M(\mathbb{Z}/p, 2) \vee S^4$. Show that $X$ and $Y$ have isomorphic cohomology rings with integral coefficients, but not with coefficients in $\mathbb{Z}/p$. (Hatcher 3.2.8)