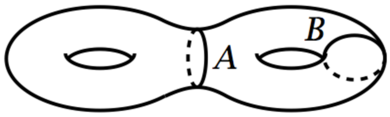
Algebraic topology
Problem sheet #3
Due: 31 Oct.

1. Compute $H_n(X, A)$ and $H_n(X, B)$ for X the orientable genus two surface and A and B as shown: (Hatcher 2.1.17b)



2. Suppose $A \subset X$ is a retract of X. Show that $H_i(X) \cong H_i(A) \oplus H_i(X, A)$.

3. Show that $H_1(X, A)$ is not isomorphic to $\tilde{H}_1(X/A)$ when $X = [0, 1]$ and $A = 0 \cup \{\frac{1}{n}\}_{n=1}^{\infty}$. (Hatcher 2.1.26)

4. Considering the Klein bottle as a union of two Möbius strips along a circle, use the Mayer–Vietoris sequence to recompute the homology of the Klein bottle.

5. Compute the homology of the space obtained by attaching a Möbius strip to $\mathbb{R}\mathrm{P}^2$ by a homeomorphism of its boundary circle to the standard $\mathbb{R}\mathrm{P}^1 \subset \mathbb{R}\mathrm{P}^2$. (Hatcher 2.2.28b)

6. The short exact sequences $0 \to C_n(A) \to C_n(X) \to C_n(X, A) \to 0$ always split, but this does not always yield splittings $H_n(X) \cong H_n(A) \oplus H_n(X, A)$ — why not? (Hatcher 2.2.27)

7. Prove the Brouwer fixed point theorem, that any continuous map $f : D^n \to D^n$ has a fixed point.

8. Show that the inclusion $i : (D^n, S^{n-1}) \to (D^n, D^n - \{0\})$ is not a homotopy equivalence of pairs, i.e. there is no map of pairs $j : (D^n, D^n - \{0\}) \to (D^n, S^{n-1})$ such that the composites ij and ji are homotopic to the identity through maps of pairs. (Hatcher 2.1.27b)