Algebraic topology
Problem sheet #2
Due: 24 Oct.

1. Show that if A is a retract of X then the map $H_n(A) \to H_n(X)$ induced by the inclusion $A \subset X$ is injective. (Hatcher 2.1.11)

2. Show that $H_1(\mathbb{R}, \mathbb{Q})$ is free abelian and find a basis. (Hatcher 2.1.18)

3. Define the reduced homology group $\tilde{H}_0(X)$ as $\ker p_*$ where $p_* : H_0(X) \to \mathbb{Z}$ is induced by the projection from X to a point. Prove that $H_0(X) \cong \tilde{H}_0(X) \oplus \mathbb{Z}$.

4. Is there a short exact sequence of the form $0 \to \mathbb{Z}/4 \to \mathbb{Z}/8 \oplus \mathbb{Z}/2 \to \mathbb{Z}/4 \to 0$? For which abelian groups A is there a short exact sequence of the form $0 \to \mathbb{Z}/p^m \to A \to \mathbb{Z}/p^n \to 0$ (p prime)? Similarly for $0 \to \mathbb{Z} \to A \to \mathbb{Z}/n \to 0$? (Hatcher 2.1.14)

5. Show that chain homotopy of chain maps is an equivalence relation. (Hatcher 2.1.12)

6. Given a short exact sequence of chain complexes $0 \to A \xrightarrow{i} B \xrightarrow{j} C \to 0$, prove that the resulting long exact sequence $\cdots \to H_i(A) \to H_i(B) \to H_i(C) \to H_{i-1}(A) \to \cdots$ is indeed exact at position B.

7. Compute the homology of the plane with k points removed.