Algebraic topology
Problem sheet #0

1. State the classification of orientable 2-dimensional manifolds. State the classification for not-necessarily orientable 2-dimensional manifolds.

2. Prove that the group structure on the fundamental group is indeed associative.

3. What is the fundamental group of the circle? Of the torus? Of the real projective plane? Of the Klein bottle?

4. Show that there is no retraction of a Möbius band onto its boundary.

5. Classify all finitely generated R-modules, for R each of $\mathbb{Z}/2$ and $\mathbb{Z}$.

6. How many distinct $\mathbb{Z}/8$-modules are there of order 4? How many distinct $\mathbb{Z}/4$-modules are there of order 8?

7. Identify the group $\mathbb{Z}/2 \oplus \mathbb{Z}/3$, identify the group $\mathbb{Z}/2 \otimes \mathbb{Z}/3$, and identify the groups $\mathrm{Hom}(\mathbb{Z}/2, \mathbb{Z}/3)$ and $\mathrm{Hom}(\mathbb{Z}/3, \mathbb{Z}/6)$.

8. Identify the cokernel of the homomorphism $f : \mathbb{Z} \to \mathbb{Z} \oplus \mathbb{Z}$ with $f(1) = (2, 2)$.